GCSE Mathematics (1MA1) – Aiming for Grade 9 3H

Student-friendly mark scheme

Please note that this mark scheme is not the one used by examiners for marking scripts. It is intended more as a guide to good practice, indicating where marks are given for correct answers. As such, it doesn't show follow-through marks (marks that are awarded despite errors being made) or special cases.

It should also be noted that for many questions, there may be alternative methods of finding correct solutions that are not shown here – they will be covered in the formal mark scheme.

Some questions require all working to be shown; in such questions, no marks will be given for an answer with no working (even if it is a correct answer).

The following table shows the marks scored on average at certain grades on similar questions from live exams.

For example: A student who achieved a Grade 8 on similar questions from either the Summer 2023 or November 2023 exam sittings achieved on average 25.4 marks from these questions.

Grade	9	8	7	6	5	4	3
Mark	25.2	15.0	9.0	4.7	2.2	0.9	0.3

Answer all questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1 (*a*) Factorise $a^2 - b^2$

$(a-b)(a+b)$

1 mark

$(a-b)(a+b)$

(1)

(*b*) Show that $2^{40} - 1$ is the product of two consecutive odd numbers.

$a^2 - b^2$ so $(2^{20})^2 - 1^2$

$= (2^{20}-1)(2^{20}+1)$ 1 mark

$2^{20} =$ even $2^{20}-1 =$ odd

so $2^{20}+1$ is next consecutive odd number.

(2)

Final mark for correct explanation   **(Total for Question 1 is 3 marks)**
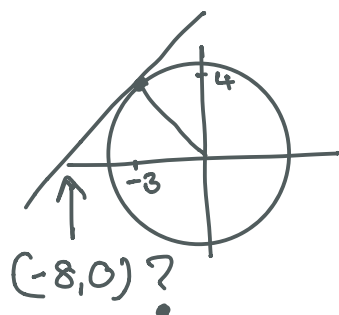
2 A circle has equation $x^2 + y^2 = 25$

The point P with coordinates $(-3, 4)$ lies on the circle.

Alex says that the tangent to the circle at P crosses the x-axis at the point $(-8, 0)$

Is Alex correct?

You must show how you get your answer.

$x^2 + y^2 = 25$ so radius = 5

$(-8, 0)$?

gradient of radius

$= \boxed{-\dfrac{4}{3}}$ 1 mark

gradient of tangent

$= \boxed{\dfrac{3}{4}}$ 1 mark

equation of tangent $\quad y = \dfrac{3}{4}x + c$

$(-3, 4)$
$\quad x \quad y$

$4 = \dfrac{3}{4} \times -3 + c$

$c = 4 + \dfrac{9}{4} \quad = 6.25$

so $\quad \boxed{y = 0.75x + 6.25}$ 1 mark

1 mark for correct conclusion

when $y = 0$

$\boxed{0.75x = -6.25 \\ x = -\dfrac{6.25}{0.75} = -8\frac{1}{3} \\ \text{so Alex is not correct.}}$

(Total for Question 2 is 4 marks)

3   Sana needs to draw the graph of $y = 3^x$ for $0 \leq x \leq 4$

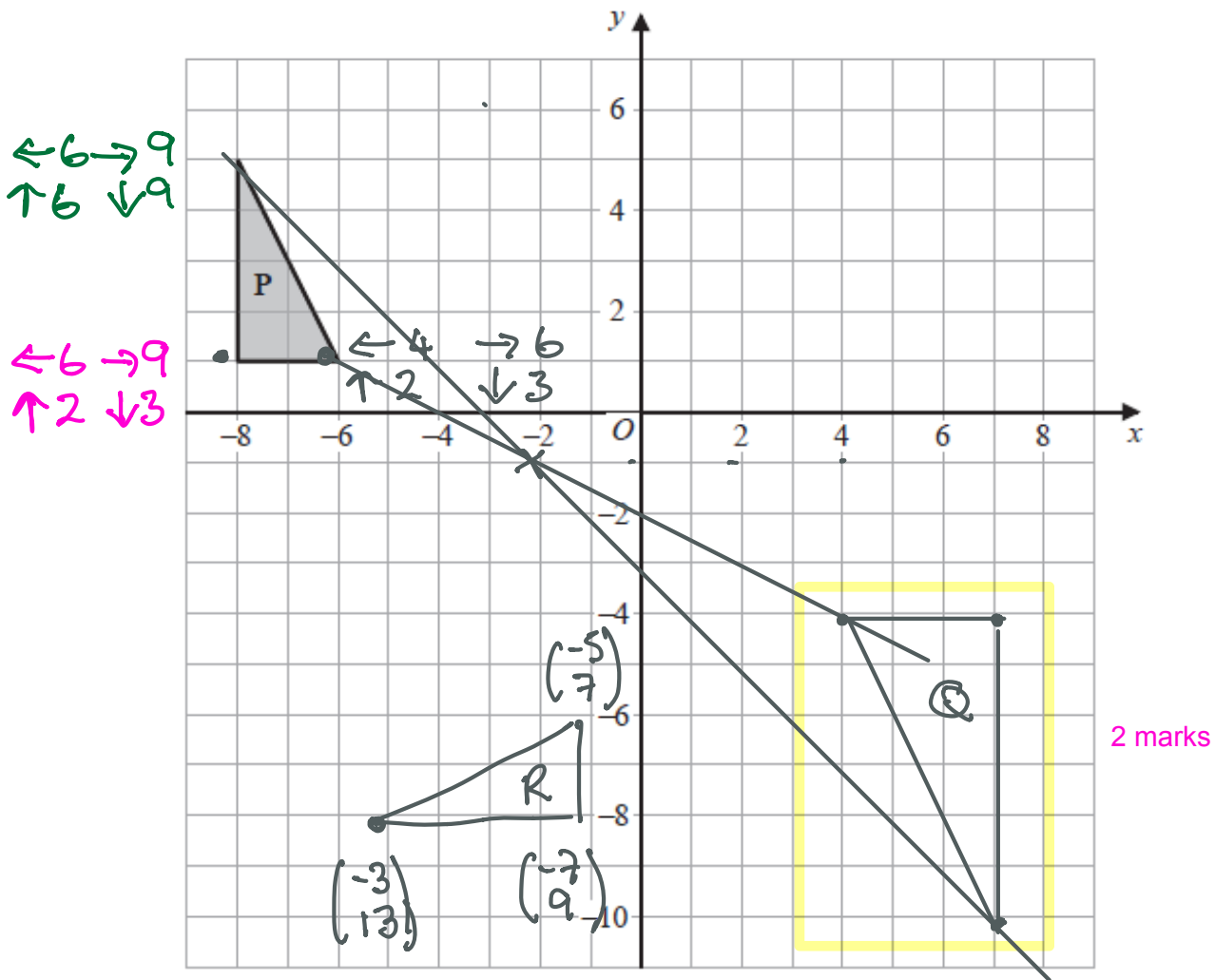She draws the graph shown on the grid.
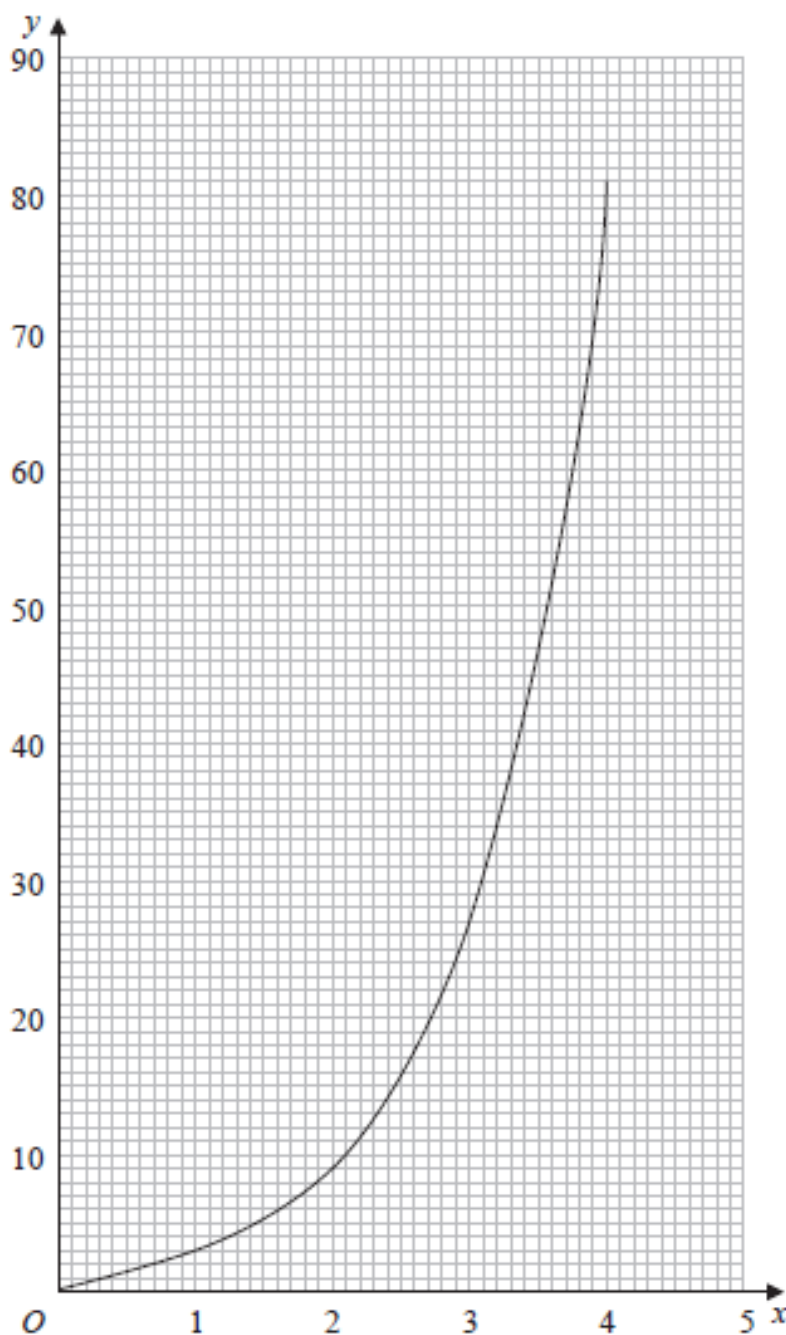
Write down one thing Sana has done wrong.

1 mark

When $x = 0$ $y = 3^0 = 1$ and not zero.
so the line should go through $(0,1)$ not $(0,0)$

(Total for Question 3 is 1 mark)

4

←6→9
↑6 ↓9

←6→9
↑2 ↓3

←4 →6
↑2 ↓3

$\begin{pmatrix} -5 \\ 7 \end{pmatrix}$

R

$\begin{pmatrix} -3 \\ 13 \end{pmatrix}$

$\begin{pmatrix} -7 \\ 9 \end{pmatrix}$

Q

2 marks

(*a*) Enlarge triangle **P** by scale factor $-1\frac{1}{2}$ with centre of enlargement $(-2, -1)$

Label your image **Q**.

(2)

Triangle **P** is transformed by a combined transformation of a rotation of 90° anticlockwise about the origin followed by a translation to give triangle **R**. Exactly one vertex of triangle **P** is invariant under the combined transformation.

1 mark for any one of these

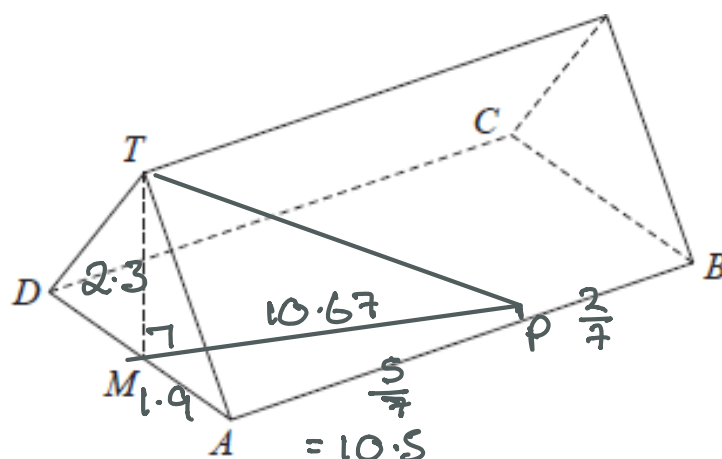(*b*) Find one possible column vector for the translation.

3 solutions possible

$\begin{pmatrix} -3 \\ 13 \end{pmatrix} \begin{pmatrix} -7 \\ 9 \end{pmatrix}$ or $\begin{pmatrix} -5 \\ 7 \end{pmatrix}$

(1)

(Total for Question 4 is 3 marks)

5   The diagram shows a triangular prism with a horizontal rectangular base *ABCD*.

M is the midpoint of *AD*.

The vertex *T* of the prism is vertically above *M*.

$$AB = 14.7 \text{ cm} \qquad BC = 3.8 \text{ cm} \quad MT = 2.3 \text{ cm}$$

P is the point on *AB* such that

$$AP : PB = 5 : 2$$

Calculate the size of the angle between *TP* and the base *ABCD* of the prism.
Give your answer correct to 1 decimal place.

$$AP = \boxed{\frac{5}{7} \times 14.7 = 10.5} \qquad \text{1 mark}$$

$$MP^2 = \boxed{\sqrt{10.5^2 + 1.9^2}} \quad \text{1 mark}$$

$$= \sqrt{113.86}$$

$$= 10.67$$

$$\boxed{\tan TPM = \frac{2.3}{10.67}} \qquad \text{1 mark}$$

$$TPM = \tan^{-1}\frac{2.3}{10.67}$$

Final mark

$$\boxed{12.2}$$

$$= 12.1638..$$

....................................°

(Total for Question 5 is 4 marks)

6 An expression for the nth term of the sequence of triangular numbers is $\dfrac{n(n+1)}{2}$

Prove that the sum of any two consecutive triangular numbers is a square number.

$$\frac{n(n+1)}{2} + \frac{(n+1)(n+1+1)}{2}$$ 1 mark

$$= \frac{n^2+n}{2} + \frac{(n+1)(n+2)}{2}$$ 1 mark

$$= \frac{n^2+n+n^2+3n+2}{2} = \frac{2n^2+4n+2}{2}$$

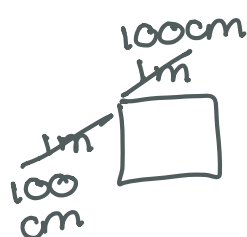$$= n^2+2n+1 = (n+1)(n+1) = (n+1)^2$$ Final mark

which is a square number.

(Total for Question 6 is 3 marks)

7 The floor plan of a house is drawn using a scale of 1 : 50
On the plan, a room in the house has a floor area of 48 cm^2
Work out the real area of the floor of this room.
Give your answer in m^2

length SF $= 50$ 1 mark

so area SF $= 50^2 = 2500$

100cm
1m

$48 \times 2500 = 120\,000$ cm^2

1m$^2 = 10,000$cm^2

1 mark

so 120000 cm$^2 \div 10000$ cm^2

$= 12$

12 Final mark

.... m^2

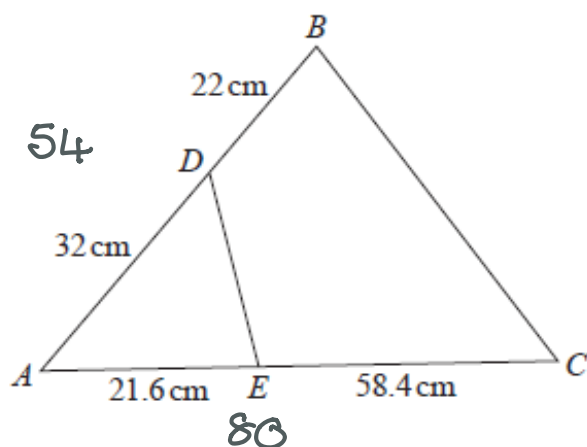**(Total for Question 7 is 3 marks)**

8 The graph of $y = f(x)$ is shown on the grid.
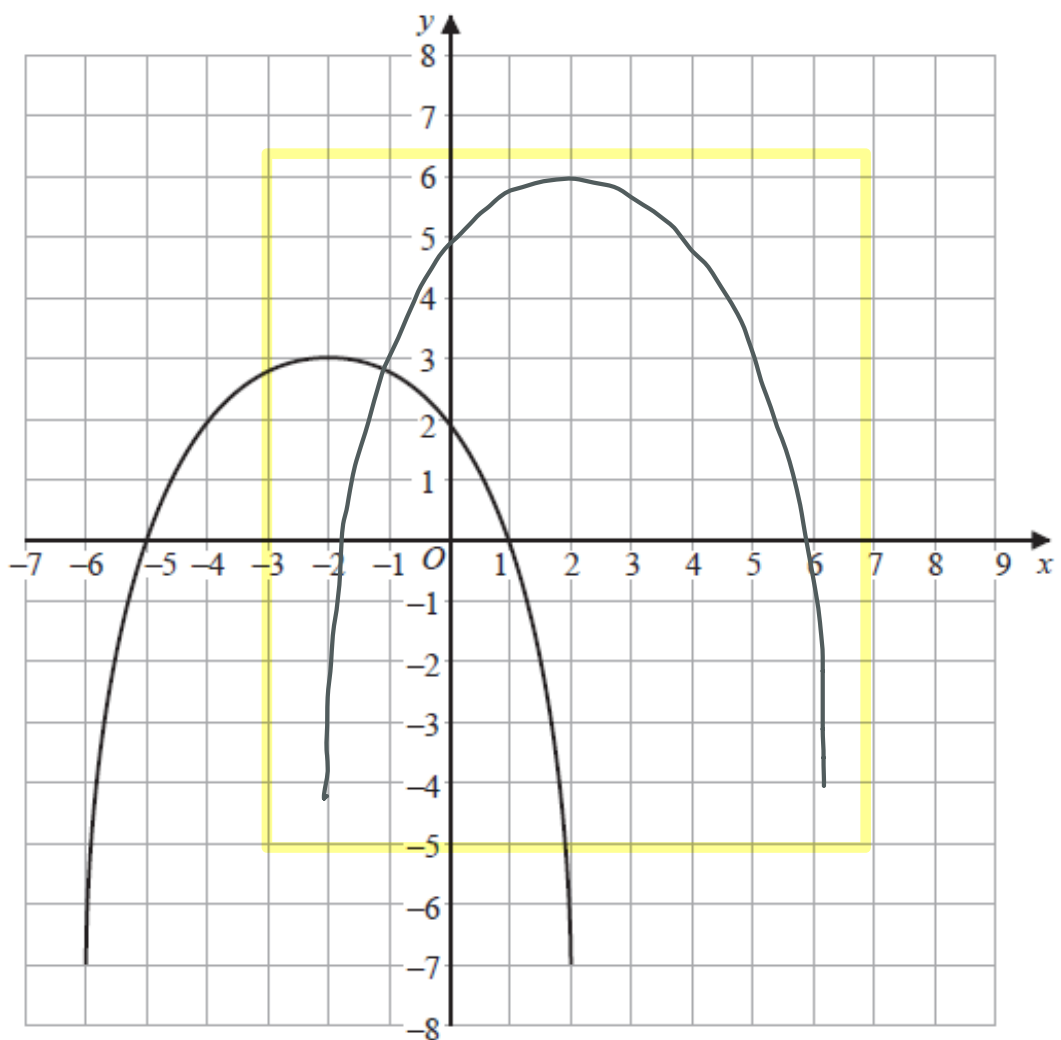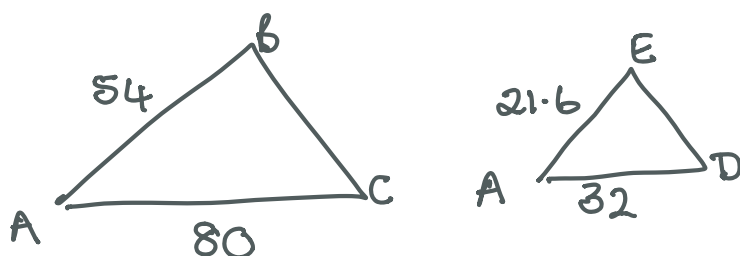
On the grid, sketch the graph of $y = f(-x) + 3$

(Total for Question 8 is 2 marks)

9   The diagram shows triangle *ABC* and triangle *AED*.

B

22 cm

54

D

32 cm

A

21.6 cm   E   58.4 cm   C

80

Show that triangle *ABC* and triangle *AED* are similar.

LOOK at the order of the letters

B

54

A

80

C

E

21.6

A   32   D

$$80 \div 32 = 2.5$$

$$54 \div 21.6 = 2.5$$

1 mark

Both triangle share angle A and the sides have a common scale factor.

1 mark for correct conclusion and statement about sharing angle A

(Total for Question 9 is 2 marks)

10 There is a total of *y* counters in a box.

There are *x* pink counters and 5 blue counters in the box.
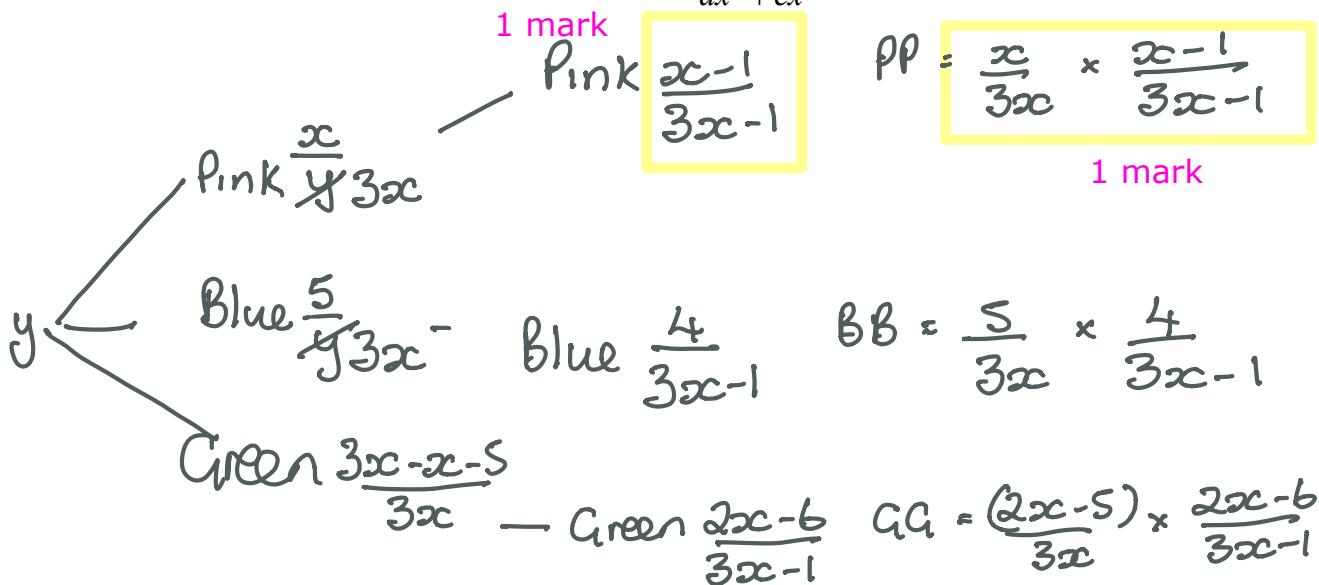
The rest of the counters are green.

$$x : y = 1 : 3$$

Freda takes at random two counters from the box.

Find, in terms of *x*, an expression for the probability that Freda takes two counters of the same colour.

Give your answer as a fraction in the form $\dfrac{ax^2 + bx + c}{dx^2 + ex}$ where *a*, *b*, *c*, *d* and *e* are integers.

Handwritten working:

1 mark

$$\frac{x}{y} = \frac{1}{3} \qquad \boxed{y = 3x}$$

Pink $\dfrac{x}{y \, 3x}$

1 mark

Pink $\boxed{\dfrac{x-1}{3x-1}}$

$$PP = \frac{x}{3x} \times \frac{x-1}{3x-1}$$

1 mark

Blue $\dfrac{5}{y \, 3x}$ — Blue $\dfrac{4}{3x-1}$ $BB = \dfrac{5}{3x} \times \dfrac{4}{3x-1}$

Green $\dfrac{3x-x-5}{3x}$ — Green $\dfrac{2x-6}{3x-1}$ $GG = \dfrac{(2x-5)}{3x} \times \dfrac{2x-6}{3x-1}$

$$P(\text{same}) = \boxed{\frac{x^2 - x}{3x(3x-1)} + \frac{20}{3x(3x-1)} + \frac{(2x-5)(2x-6)}{3x(3x-1)}}$$

1 mark

$$= \frac{x^2 - x + 20 + 4x^2 - 12x - 10x + 30}{9x^2 - 3x}$$

$$= \frac{5x^2 - 23x + 50}{9x^2 - 3x} \qquad \boxed{\frac{5x^2 - 23x + 50}{9x^2 - 3x}}$$

Final mark

(Total for Question 10 is 5 marks)

11 Ebony makes some bracelets to sell.

The materials to make all the bracelets cost £190, correct to the nearest £5

Ebony sells all the bracelets for a total of £875, correct to the nearest £5

The total time taken to make and sell all these bracelets was 72 hours, correct to the nearest hour.

Ebony uses this method to calculate her hourly rate of pay

$$\text{Hourly rate of pay} = \frac{\text{total selling price} - \text{total cost of materials}}{\text{total time taken}}$$

The minimum hourly rate of pay for someone of Ebony's age is £8.20

By considering bounds, determine if Ebony's hourly rate of pay was definitely more than £8.20

You must show all your working.

materials   190 nearest £5 → 192·50
                              → 187·50

sells   875 nearest £5 → 877·50
                              → 872·50

time   72 hours nearest hour → 72·5
                                    → 71·5

1 mark for any one of these bounds

Looking for LB of hourly rate of pay.

$$= \frac{872·50 - 192·50}{72·50}$$

1 mark

1 mark

$= 9·3751...$ so £9·38

Yes it is definitely more than £8.20

Final mark

(Total for Question 11 is 4 marks)

12 Given that the vector $a\begin{pmatrix}2\\6\end{pmatrix}+b\begin{pmatrix}8\\2\end{pmatrix}$ is parallel to the vector $\begin{pmatrix}13\\6\end{pmatrix}$

find an expression for b in terms of a.

gradient $=\dfrac{6}{13}$

1 mark
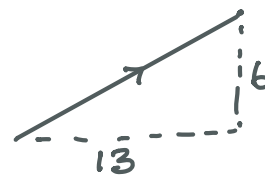
① $6a+2b = 6$ × 4

② $2a+8b = 13$

③ $24a + 8b = 24$

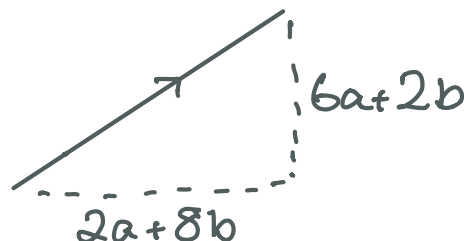③ − ② $\quad 22a \qquad = 11$

$a = 0.5$ 1 mark

subn ② $\quad 1 + 8b = 13$

$8b = 12$

$b = \dfrac{3}{2} = 1.5$

$6a+2b$

$2a+8b$

$a = 0.5 \quad b = 1.5$

so $b = 3a$

Final mark

$b = 3a$

(Total for Question 12 is 3 marks)

TOTAL FOR PAPER IS 37 MARKS